JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

1ST YEAR 2ND SEMESTER 2016/2017 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: SMA 105

COURSE TITLE: INTRODUCTION TO PROBABILITY THEORY

EXAM VENUE: **STREAM: (Bed. Arts, Bed Science, Bed Arts (Special Needs))**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Outline FIVE axioms of probability (5 Marks)

b) A family has two children. What is the conditional probability that both are boys given that atleast one of them is a boy? (3 Marks)

c) Consider two urns. The first contains two white and seven black balls and the second contains five white and six black balls. We flip a fair coin and then draw a ball from the first first urn or the second urn depending on whether the outcome was head or tail. What is the conditional probability that the outcome of the toss was head given that a white ball was selected? (4 Marks)

d) Define the following terms (4 Marks)

   i. Random variable
   ii. Bernoulli random variable
   iii. Exponential random variable
   iv. Gamma random variable

e) A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of three is taken by the inspector. Find the expected value of the number of good components in this sample. (7 marks)

f) Find $E\left(Y/X\right)$ for the density function

$$f(x,y)=\begin{cases}\frac{1+3y^2}{4} & 0\le x\le 2; 0\le y\le 1\\ 0 & Otherwise\end{cases}$$

(3 Marks)

g) Let X be a random variable having the density function given by

$$f(x)=\begin{cases}x^2/3 & -1<x<2\\ 0 & Otherwise\end{cases}$$

Find the variance of the random variable $g(x)=4x+3$ (4 Marks)

QUESTION TWO (20 MARKS)

a) Show that variance of a random variable X is given by $\sigma^2=E\left(X^2\right)-\mu^2$ (4 Marks)

b) Let the random variable X represent the number of automobiles that are used for official business purposes on any given workday. The probability distribution for company A is

X	1	2	3
$f(x)$	0.3	0.4	0.3

And for company B is

X	0	1	2	3	4
$f(x)$	0.2	0.1	0.3	0.3	0.1

Show that the variance of the probability distribution for company B is greater than that of company A. (10 Marks)

c) If X is uniformly distributed over (0,10), calculate the probability that

i. $X < 3$

ii. $X > 7$

iii. $1 < x < 6$ (6 Marks)

QUESTION THREE (20 MARKS)

a) Let X and Y be random variables with joint probability distribution $f(x, y)$. Show that the covariance of X and Y is $\sigma_{XY} = E(XY) - \mu_X \mu_Y$ (5 Marks)

b) The fraction X of male runners and the fraction Y of female runners who complete in marathon races are described by the joint density function

$$f(x, y) = \begin{cases} 8xy & 0 \le y \le x \le 1 \\ 0 & Otherwise \end{cases}$$

Find the covariance of X and Y (15 Marks)

QUESTION FOUR (20 MARKS)

Calculate the moment generating functions of the following

i. Binomial distribution (10 Marks)

ii. Poisson distribution (10 Marks)

QUESTION FIVE (20 MARKS)

a) Let X be uniformly distributed over (0, 1). Calculate $E[X^3]$. (8 Marks)

b) Let X be normally distributed with parameters μ and σ^2. Find $\text{var}(X)$ (8 Marks)

c) Calculate $\text{var}(X)$ when X represents the outcome when a fair die is rolled. (4 Marks)