JARAMOGI OGINGA ODINGA UNIVERSITY

OF SCIENCE & TECHNOLOGY

UNIVERSITY EXAMINATIONS 2012/2013

2ND YEAR 1ST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE ACTUARIAL SCIENCE

(REGULAR)

COURSE CODE: SAS 303

COURSE TITLE: ESTIMATION THEORY

DATE: 12/8/2013 **TIME: 9.00-11.00 AM**

DURATION: 2 HOURS

INSTRUCTIONS

1. **This paper consists of 5 Questions.**
2. **Answer Question 1 (Compulsory) and any other 2 questions.**
3. **Write your answers on the answer booklet provided.**

QUESTION ONE (20 MARKS)

a) Distinguish clearly the terms sufficiency and completeness as used in estimation theory. (4marks)

b) Consider the probability density function $f(x) = c(1 + \theta x) \quad -1 \leq x \leq 1$
For the true value of c, find the MME of θ (5marks)

c) Let $X_1, X_2, \dots, X_m$ be iid binomial (n,P) random variables. Find the Cramer-Rao lower bound for $d(p) = p^2$ (6marks)

d) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Show that $\bar{X}^2$ is biased for μ^2 and state the amount of bias. (4marks)

e) Suppose we have a random sample of size 2n from a population denoted by $X$and $(X) = \mu$, Var (X) = σ^2. Let $\bar{X}_1 = \frac{1}{2n}\sum_{i=1}^{2n}(X_i)$, $\bar{X}_2 = \frac{1}{n}\sum_{i=1}^{n}(X_i)$ be two estimators of μ. Which is the better estimator of μ. (5marks)

f) It is largely thought that the average daily intake of dairy products differ significantly between the urban male and female. A survey of 50 urban men and women yielded the following results.

gender	male	female
Sample mean	756	762
Sample standard deviation	35	30

Based on both a 95% and a 99% confidence intervals accept or refute the above claim. (6marks)

QUESTION TWO (20 MARKS)

a) Let $X_1, X_2, \dots, X_n$ be iid random variables from the uniform $u(x, \theta)$distribution. Show that

$$T_n = \left(\prod_{i=1}^{n} X_i\right)^{1/n}$$

is a consistent estimator of θe^{-1} (8marks)

b) Use the Lehmann Scheffe method of construction of minimal sufficient statistics to find the minimal sufficient statistic for $\theta = (\mu, \sigma^2)$ given $X_1, X_2, \dots, X_n$ are iid random variables from $N(\mu, \sigma^2)$ (7marks)

c) Let X be gamma random variable with probability density function

$$f(x, \theta) = \begin{cases} \dfrac{x^{p-1}e^{-x/\theta}}{\theta^p \Gamma(p)}, & x > 0, p > 0, \theta > 0 \\ 0, & x \leq 0 \end{cases}$$

Show that $f(x, \theta)$ belongs to a 1- parameter exponential family whenever p is known. (5marks)

QUESTION THREE (20 MARKS)

a) Let $X_1, X_2, \dots, X_n$ are iid random binomial (1,P) random variables. A biased coin is tossed n times with probability of success as P. Show that to estimate P it is sufficient to know the statistic $T = \sum_{i=1}^{n} X_i$ (6marks)

b) Let $X_1, X_2, \dots, X_n$ are iid $N(\mu, \sigma^2)$ random variables. Find the UMVUE of

i. μ

ii. σ^2

iii. $d(\mu) = \mu^2$ (9marks)

c) Based on the data below, obtain estimates of the UMVUE established for the three parameters in b above.

observation	3	5	8	10	13	17	22	26
frequency	3	8	10	12	7	5	3	2

(5marks)

QUESTION FOUR (20 MARKS)

a) Suppose

$$f(x,P,\theta) = \begin{cases} \dfrac{\theta^p x^{p-1} e^{-\theta X}}{\Gamma(p)}, & x > 0, p > 0, \theta > 0 \\ 0, & otherwise \end{cases}$$

Derive $E(X)$ and $E(X^2)$ hence propose the MME for θ and P. (9marks)

b) Let $X_1, X_2, \dots, X_n$ be iid poisson random variables with $f(x_i,\theta) = \frac{e^{-\theta}\theta^{x_i}}{x_i!}$ $x = 0,12, \dots$ Obtain Fisher's information for estimation of θ hence give the C.R.L.B for estimation of $d(\theta) = \theta^2$. (5marks)

c) Two insurance companies are selling a new product that targets the middle class and the elite in society. A random survey yielded the following information on the attitude of the intended market towards this product.

	Middle class	elite
Sample size	50	100
Number favoring new product	38	65

i. Estimate the difference in the true proportions favoring the new product with a 99% confidence interval. (4marks)

ii. If both samples are pooled into one. Find a point estimate for the proportion that favors the new product and give the margin of error. (2marks)

QUESTION FIVE (20 MARKS)

a) Let $X \sim N(\mu_0, \sigma^2)$. Find a complete sufficient statistic for σ^2. (5marks)

b) Let $X_1, X_2, \dots, X_n$ be a random sample for a binomial random variable X with parameter (m,p) where m is assumed to be known and p unknown. Derive the general MLE of p hence an exact estimate given $\sum_{i=1}^{n} x_i = 60$, $n = 30$, $m = 10$ (7marks)

c) Two random samples on the average score on an aptitude test were tabulated as shown

Sample A		Sample B	
score	frequency	score	frequency
1	4	1	2
2	6	2	3
3	5	3	4
4	3	4	6
5	6	5	5

6	6	6	5
7	4	7	3
8	3	8	6
9	2	9	4
10	1	10	2

Obtain a 90% confidence interval for the difference of means and comment on it. (8marks)