JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR EXAMINATION FOR DEGREE IN BED SCI. AND BED ARTS
SPECIAL RESIT EXAM 2020/2021
MAIN CAMPUS (BONDO)

COURSE CODE: **SMA 403**
COURSE TITLE: **TOPOLOGY I**
EXAM VENUE:
STREAM: **BED SCI. AND BED ARTS**

DATE:……………………..EXAM SESSION …………………..

TIME: **2 HOURS**

Instructions:

1. **Answer all questions in Section A and any other 2 questions in Section B**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

SECTION A COMPULSORY (30 MARKS)

QUESTION ONE (30 MARKS)

a) Define the following terms: (6 marks)

i) Metric space

ii) Topological space

b) Prove that $(A \bigcup B)^c = A^c \bigcap B^c$ (4 marks)

c) Given the set $U = \{1,2,3,\ldots,9\}$, $A = \{2,4,6,8\}$ and $B = \{1,3,7,9\}$. Find (3 marks)

i) $A \Delta B$

ii) $A \bigcup B$

iii) A^c

d) Prove that the empty set is unique (3 marks)

e) Show that the function $d : XxX \to R$ defined as $d(x,y) = \{^{0, x=y}_{1, x \neq y}$ $\forall x, y \in X$ is a metic on X. (8 marks)

f) Prove that the union of arbitrary family of open sets is open. (6 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) Given $X = \{1, 2, 3, 4, 5\}$ $\tau = \{X, \emptyset, \{5\}, \{2,3\}, \{2,3,5\}, \{1,2,3,4\}\}$ $A = \{5\}$, $B = \{3, 4\}$

Find (i). $\overline{A}$ (2 marks)

(ii). Boundary of B (4 mark)

(iii). Limit points of A. (4 marks)

b) Let (X, τ) be a topological space and $A \subseteq X$. Prove that A is closed if and only if it contains all its limit points. (10 marks)

QUESTION THREE (20 MARKS)

a) Let $X = \{1,2,3\}$ and $\tau = \{X, \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$. Show that τ is a topology on X. (5 marks)

b) Let X be a non-void set and τ_1, τ_2 be topologies on *X*. Show that $\tau_1 \bigcap \tau_2$ is a topology on X. (5 marks)

c) Let $X = \{1,2,3\}$ $\tau = \{\emptyset, X, \{1\}, \{1,2\}, \{2,3\}\}$ and $A = \{1,3\}$. Find the Int*(A).* (5 marks)

d) Show that every convergent sequence in T_2-space converges to a unique limit. (5 marks)

QUESTION FOUR (20 MARKS)

a) Show that the indiscrete topological space is T_0 space. (4 marks)

b) Prove that the topological space (X, τ) is a T_1space if and only if every singleton subset of X is closed. (6 marks)

c) Show $d: RxR \rightarrow R$ defined by $d(x, y) = |x^2 - y^2|$ is a metric on R. (5 marks)

d) Let $X = \{1,2,3,4,5\}$, $\tau = \{X, \emptyset, \{5\}, \{2,3\}\{2,3,5\}, \{1,2,3,4\}\}$ and $A = \{3,4,5\}$. Find all the accumulation points of A. (5 marks)

QUESTION FIVE (20 MARKS)

a) Define a topology and suppose that $X = \{a, b, c, d, e, f\}$ and $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e, f\}\}$, Is τ a topology on X? (5 marks)

b) Let (X, τ) be a topological space and$A \subseteq X$. If $X = \{1,2,3,4\}$, $A = \{1,2\}$, $B = \{3,4\}$,

$$\tau = \{\phi, \ \{1,2\}, \{1,2,3\}, \{1,2,4\}, X\ \}.$$

Find

(i). Int(A) (1 mark)

(ii). Ext(A) (2 marks)

(iii). Bdy(A) (2 marks)

(iv). Bdy(B) (5 marks)

c) Show that the indiscrete topological space is indeed a topological space. (5 marks)