JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

2021/2022 SPECIAL RESIT EXAMINATIONS

MAIN REGULAR

COURSE CODE: SPH 303
COURSE TITLE: QUANTUM MECHANICS I
EXAM VENUE: **STREAM: EDUCATION**
DATE: **EXAM SESSION:**
TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants

$\hbar = 1.054x10^{-34}Js$

$1eV = 1.6x10^{-19}J$

$h = 6.63x10^{-34}Js$

$Mass of electron = 9.1x10^{-31} kg$

$Mass of proton = 1.672x10^{-27} kg$

SECTION A

QUESTION 1(30 MARKS)

a)

i. Calculate the **de Broglie** wavelength for an electron having kinetic energy of $1eV$.

(3 marks)

ii. In the **double-slit experiment**, two waves defined by $\psi_1 = \frac{1}{\sqrt{2}} e^{ix}$ and $\psi_2 = e^{ix}$ pass through the slits. Determine the probability density on the screen.

(4 marks)

b) Explain the **probabilistic interpretation** of quantum mechanics.

(2 marks)

c) Derive the **time-independent** Schrödinger equation. **(4 marks)**

d) Define the following terms as used in quantum mechanics.

i. Bound state. **(1 mark)**

ii. Tunnelling. **(1 mark)**

e) The **expectation value** of the position of a particle described by the wave function $\psi = \frac{1}{2}x$ limited to the x-axis between $x = 0$ and $x = b$ is $\frac{1}{16}$. Find the value of b.

(3 marks)

f) Use the de Broglie relation to show that the allowed electron energy states in the hydrogen atom can be written as

(4 marks)

g) An **eigenfunction** of the operator $\frac{d^2}{dx^2}$ is $\psi = e^{2x}$. Find the corresponding **eigenvalue**.

(3 marks)

h) An operator is defined by $\hat{D}_x = \frac{\partial}{\partial x}$. Determine the **Heisenberg's uncertainty product** in the measurement of $\hat{x}$ and $\hat{D}_x$. **(4 marks)**

i) State **ONE** postulate of Quantum mechanics. **(1 mark)**

SECTION B

Answer any TWO questions in this section.

QUESTION 2 (20 MARKS)

a) Solve the one –dimensional time-independent Schrödinger equation for a particle in a **step potential with a step height** greater than the total energy.

(15 marks)

b) Estimate the **penetration distance** for a very small dust particle of mass $5x10^{-14} kg$ moving at a velocity of $0.02 ms^{-1}$ if the particle impinges on a potential step of height twice its kinetic energy. **(5 marks)**

QUESTION 3 (20 MARKS)

a) Show that the general **wave function for a free particle** in one-dimensional motion is given by $\psi(x) = acos\frac{1}{\hbar}(px + \hbar\theta)$ where the symbols have their usual meanings.

(16 marks)

b) Show that the constant E in the time-independent Schrödinger equation is the **expectation** value of the Hamiltonian. **(4 marks)**

QUESTION 4 (20 MARKS)

a) A measurement establishes the position of a proton with an accuracy of $1.00x10^{-11}m$. Use **Heisenberg's uncertainty principle** to find the uncertainty in the proton's position $1.00s$ later. **(9 marks)**

b) The Hamiltonian of a quantized **linear harmonic oscillator** is given by

$+a+\frac{1}{2}$

$\hat{a}$ **.** Obtain an expression for the energy at second energy level.

$\hat{H} = \hbar\omega$

(7 marks)

c) Calculate the frequency of a **harmonic oscillator** at ground state if the ground state energy is $3.4eV$. **(4 marks)**

QUESTION 5 (20 MARKS)

a) Consider a particle in **an infinite potential** box (potential zero inside the box and infinity outside) which extends from $x = 0$ to $x = \pi$Å. Obtain an expression for the energy eigenvalues in terms of n where $n = 1,2, ...$

(9 marks)

b) Suppose an electron has a wave function $\psi(x) = cx^3e$ $-\alpha \vee x \vee$ where α is a constant.

i. Find the constant c that ensures the given wave function is properly **normalized**. You may use the standard integral

$$\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}}$$ **(6 marks)**

ii. Find the **expectation** value of x**.** **(5 marks)**