JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL SCIENCE

4TH YEAR 2ND SEMESTER 2024/2025

REGULAR (MAIN)

COURSE CODE: WAB 2404

COURSE TITLE: COMPUTATIONAL FINANCE.

EXAM VENUE:LAB 13 **STREAM: (BSc Actuarial Science)**

DATE:14/4/25 **EXAM SESSION:9-11.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE 30 MARKS

a. Define

i.) Intrinsic value.

(2 marks)

ii.) Write down the intrinsic value of a put option at time t.

(2 marks)

b. Suppose that the price of Share X is 112 and that a put option on Share X with an exercise price of 110 is currently priced at 5.
Calculate the intrinsic value and time value of the option.

(2 marks)

c. Given a filtered probability space (Ω,F,F_t,P), what are the conditions for a stochastic process X_t is called a martingale with respect to the filtration, F_t?

(3marks)

d. Show that $Var(Z_{t+u} - Z_t = u)$ for u > 0

(5 marks)

e. Find the stochastic differential equation for $W_t^{2.}$

(2marks)

f. i. Write down Ito's Lemma as it applies to a function $f(X_t)$ of a stochastic process X_t that satisfies the stochastic differential equation $dX_t = \sigma_t dB_t + \mu_t dt$, where B_t is a standard Brownian motion.

(2marks)

ii. Hence find the stochastic differential equations for each of the following processes:

(a) $G_t = \exp(X_t)$

(b) $Q_t = X_t^2$

(4marks)

g. State and explain two basic types of options.

(4marks)

h. A fixed-interest security pays coupons of 8% pa half-yearly in arrear and is redeemable at 110%. Two months before the next coupon is due, an investor negotiates a forward contract to buy £60,000 nominal of the security in six months' time. The current price of the security is £80.40 per £100 nominal and the risk-free force of interest is 5% pa.
Calculate the forward price.

(4 marks)

QUESTION TWO 20 MARKS

a. State and explain the properties of a Wiener process.

(5 marks)

b. $\{X_t\}$ be a continuous-time stochastic process defined by the equation
$X_t = \alpha W_t^2 + \beta$, where

$\{W_t\}$ is a standard Brownian motion and α β and are constants.
By applying Ito's Lemma, or otherwise, write down the stochastic differential equation satisfied by X_t .

(5marks)

c. Assume that the spot rate of interest at time t ,S(t), can be modelled by $S(t)= e^{-2\mu W(t)}$ where
W(t) is a Brownian motion with drift coefficient μ and volatility coefficient 1 such that W(0) = 0.

(i) Write down an expression for W(t) in terms of a standard Brownian motion, B(t).

(2marks)

(ii) Show that {S(t): t>0} is a continuous-time martingale.

(8marks)

QUESTION THREE 20 MARKS

a. Let t W_t be a standard Brownian motion. Prove that $Z_t = Z_0 + \sigma W_t + \mu t$ is a Brownian motion with diffusion coefficient σ and drift μ.

(5 marks)

b. Consider an American put option on a non-dividend-paying share.
List the five factors that determine the price of this option and, for each factor, state whether an increase in its value produces an increase or a decrease in the value of the option.

(5marks)

c. Let X be an Ito process that satisfies

$$dX_t=\mu(X_t,t)dt + \sigma(X_t,t)dB_t$$

where B_t is a standard Brownian motion. Let $f(X_t,t)$ be a function of t and X_t .

(i) By considering Taylor's theorem, suggest a partial differential equation that must be satisfied by $f(X_t,t)$ in order that it is a martingale.

(5 marks)

(ii) Verify that your equation holds when $f(X_t,t) = B_t^2-t$.

(5 marks)

QUESTION FOUR 20 MARKS

a. Let S_t be a geometric Brownian motion process defined by the equation
$S_t=\exp(\mu t+\sigma W_t)$,
where W_t is a standard Brownian motion and μ and σ are constants.

(i) Write down the stochastic differential equation satisfied by $X_t= \log_e S_t$. (2marks)

(ii) By applying Ito's Lemma, or otherwise, derive the stochastic differential equation satisfied by S_t . (3marks)

(iii) The price of a share follows a geometric Brownian motion with μ=0.06 and σ=0.25 (both expressed in annual units). Find the probability that, over a given one-year period, the share price will fall.

(5marks)

b. Derive the Black-Scholes equation.

(10 marks)

QUESTION FIVE 20 MARKS

a. State the assumptions underlying the Black-Scholes model.

(5marks)

b. An investor buys, for a premium of 187.06, a call option on a non-dividend-paying stock whose current price is 5,000. The strike price of the call is 5,250 and the time to expiry is 6 months. The risk-free rate of return is 5% pa continuously compounded.

The Black-Scholes formula for the price of a call option on a non-dividend-paying share is assumed to hold.

(i) Calculate the price of a put option with the same time to maturity and strike price as the call.

(5marks)

(ii) The investor buys a put option with strike price 4,750 with the same time to maturity. Calculate the price of the put option if the implied volatility were the same as that in (i). [You need to estimate the implied volatility to within 1% pa of the correct value.]

(10marks)