JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

1st YEAR 2nd SEMESTER 2016/2017 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 822

COURSE TITLE: BANACH ALGEBRA I

EXAM VENUE: **STREAM: (Msc. Pure Mathematics)**

DATE: **EXAM SESSION: TWO**

TIME: 3.00 HOURS

Instructions:

1. **Answer any THREE questions only**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE [20 MARKS]

a) Define multiplicative linear functional ϕ and show that $\|\phi\| = 1$. (7 marks)

b) Define a C^*-algebra giving two examples and show that a linear functional f is bounded with $\|f\| = f(e)$. (9 marks)

c) Describe a Gelfand-Naimark-Segal transform and hence show that it is a contractive Banach algebraic homomorphism. (3 marks)

QUESTION TWO [20 MARKS]

a) Analytically describe: A normed* -algebra; sub-algebra and Banach algebra. (6 marks)

b) Prove that the intersection of two Banach algebras is Banach algebra. (10 marks)

c) State and prove the condition under which $l^{\infty}(S)$ the set of all bounded valued on nonempty set *S* is a unital Normed *- algebra. (4 marks)

QUESTION THREE [20 MARKS]

a) Differentiate between the left inverse and right inverse in Banach algebras. (6 marks)

b) Show that an inverse in a Banach algebra is unique. (7 marks)

c) Let Ω be a Banach algebra and e its unity. Prove that if $x \in \Omega$ and $\|ex\| \prec 1$ then there exist x^{-1} such that $\|x^{-1}\| \leq \frac{e}{e\|ex\|}$. (10 marks)

QUESTION FOUR [20 MARKS]

a) Describe: Trivial ideal, modula ideal, Maximal ideal and prime ideal. (8 marks)

b) Let $C_o(\Omega)$ be an algebra and $M_w = \{f \in C_o(\Omega): f(w) = 0\}$. Show that M_w is a modula ideal. (8 marks)

c) Describe the process of unitization of normed algebras. (8 marks)

QUESTION THREE [20 MARKS]

a) Prove that if *Z* is the set off all intergers with counting measure then $l^1(Z)$ is a Banach algebra. (8 marks)

b) Prove that any nonempty open subset of irreducible Banach algebra is dense and irreducible. Moreover, prove that if *Y* is a subset of a Banach algebra *X*, which is irreduciblein its induced sub-algebra then the closure of *Y* is also irreducible. (12 marks)