JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

1st YEAR 2nd SEMESTER 2016/2017 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 820

COURSE TITLE: OPERATOR THEORY I

EXAM VENUE: **STREAM: (Msc. Pure Mathematics)**

DATE: **EXAM SESSION: TWO**

TIME: 3.00 HOURS

Instructions:

1. **Answer any THREE questions only**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE [20 MARKS]

a) Define: A normal operator, Hyponormal operator and Hermitian operator. (6 marks)

b) State and prove the spectral theorem for compact self-adjoint operator. (14 marks)

QUESTION TWO [20 MARKS]

a) Define the terms: Non-invariant subspace and reducing subspace. (6 marks)

b) Describe orthogonal direct sum and product of subspaces. (4 marks)

c) Show that an operator is self-adjoint iff $\langle Ax, x\rangle \in R, \forall x \in H$. (10 marks)

QUESTION THREE [20 MARKS]

a) Differentiate between projection and idempotent giving two example s in each case in Hilbert spaces. (4 marks)
b) Let P be a nonzero projection. Prove that if P is positive it is an idempotent. (8 marks)
c) Prove that an operator is normal if and only if it is self-adjoint. (8 marks)

QUESTION FOUR [20 MARKS]

a) Define: left shift operator, adjoint of an operator and contractive operator. (6 marks)
b) Let *H* and *K* be Hilbert spaces and let $A: H \to K$ be bounded operator. Prove that there exist $A^* : K \to H$ which is bounded and linear. (14 marks)

QUESTION FIVE [20 MARKS]

a) Describe the process of diagonalization of self-adjoint complex matrix operators. Give three example to show how diagonalization is carried out. (6 marks)
b) Let $A: H \to H$ be compact and $\lambda \neq 0$ be an eigenvalue of *A*. Prove that Ker($A - \lambda I$) is finite dimensional. (14 marks)