JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THEDEGREE OF BACHELOR OF EDUCATION (SCIENCE)

3RD YEAR 1ST SEMESTER

MAIN

REGULAR

COURSE CODE: SPH 313

COURSE TITLE: CLASSICAL MECHANICS

EXAM VENUE: **STREAM: (BED SCI)**

DATE: **EXAM SESSION:**

TIME: 2:00HRS

Instructions:

1. **Answer question 1 (Compulsory) in Section A and ANY other 2 questions in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a. Three blocks of masses m_1= 28kg, m_2 =40 kg and m_3=80kg are connected by two light inelastic strings that passes over a frictionless pulleys, as shown in Figure 1. M_2 is sliding on a rough plane whose coefficient of friction is 0.35.

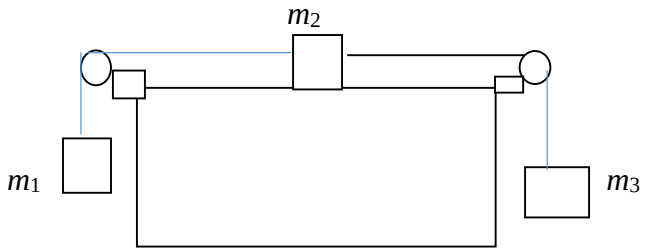

Figure 1

Find the common acceleration of the three blocks and the tensions on the two strings.

(5 marks)

b. Show that the total kinetic energy of a system of many particles is the sum the kinetic energy of the Centre of Mass motion and the kinetic energy relative to the Centre of Mass. (5 marks)

c. Using the Lagrangian formulation, obtain the equations of motion for a particle of mass ***m*** suspended on a spring pendulum of length ***l.*** (5 marks)

d. Derive the Hamiltonian equation of a system hence or otherwise obtain the hamiltonian of a free particle moving in one direction ***x*** and described in a uniform frame being accelerated by acceleration ***a.*** **(5 marks)**

e. Present the analytical concept of **twin paradox** (6 marks)

f. Briefly explain the concept of time dilation and length contraction with reference to theory of relativity (4 marks)

QUESTION TWO (20 Marks)

a. Mass M_1 is held on a plane with inclination angle θ to the horizontal, and mass M_2 hangs freely vertically over the side. The two masses are connected by a massless string which runs over a massless pulley. The coefficient of kinetic friction between M_1 and the plane is μ. M_2 is released from rest.

Assuming that M_2 is sufficiently large so that M_1 gets pulled up the plane, Determine

i) The common acceleration of the masses

ii) The tension in the string

QUESTION THREE (20 Marks)

a. The shell theorem states that a uniform shell of matter attracts an external particle as if all the shell's mass were concentrated at its center. Give the mathematical proof of this theorem (8 marks)

b. A double pendulum consists of two masse m_1 and m_2. The length of the string supporting m_1 is l_1 while the length of the string from mass m_1 to m_2 is l_2. m_1 is inclined to the vertical at θ while m_2 is inclined at α *to the vertical*

i.) Obtain the Lagrangian of the system (6 marks)

ii.) Obtain the equations of motion to the system (6 marks)

QUESTION FOUR (20 Marks)

a. Show that the shortest path between two points in a plane is a straight line. (6 marks)

b. A bead is released from rest at the origin and slides down a frictionless wire that connects a point *(x,y)* on the plane to the origin (0,0). You wish to shape the wire so that the bead reaches the endpoint in the shortest possible time. Let the desired curve be described by the function *y(x),* with downward taken to be positive.

i) Show that *y(x)* satisfies

$$1+y'^2=\frac{B}{y}\ ,$$ where B is a constant. (8 marks)

ii) Show that x and y may be written as

$x=a(\theta-\sin\theta),\qquad y=a(1-\cos\theta).$

(6 marks)

QUESTION FIVE (20 Marks)

a. A clock starts on the ground and then moves up a tower at constant speed v. It sits on top of the tower for a time T and then descends at constant speed v. If the tower has height h, how long should the clock sit at the top so that it comes back showing the same time as a clock that remained on the ground? (10 marks)

b. A spaceship travels at speed v to a distant star. Upon reaching the star, it decelerates and then accelerates back up to speed v in the opposite direction (uniformly, and in a short time compared with the total journey time). By what fraction does the traveler age less than her twin on the earth? (Ignore the gravity from the earth.)

Work in:

(a) The earth frame.

(b) The spaceship frame (10 marks)