JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION SCIENCE, EDUCATION ARTS AND SPECIAL EDUCATION

1ST YEAR 2ND SEMESTER 2024/2025 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2109

COURSE TITLE: INTRODUCTION TO PROBABILITY AND DISTRIBUTION THEORY

EXAM VENUE:LAB-AUD/LAB 1/2 **STREAM: BED**

DATE: 14/4/25 **EXAM SESSION: 9-11.00 AM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) Suppose an urn contains 5 black balls and 4 white balls. 3 balls are drawn from the urn without replacement. What is the probability that the last ball drawn is black given the the first ball drawn was white? (5 Marks)

b) Two events A and B are such that $P(A) = 0.4, P(B/A) = 0.35, P(A/B) = 0.28$. Obtain $P(A \cup B)$ (4 Marks)

c) A student is preparing for his entrance examinations which he must pass. The student can take the examination physically or online. Probabilities are 0.7 for a physical examination and 0.8 that he will pass given he does the exam physically. His chances of passing if he does the examination online is 0.65. Find the probability that the student will pass the entrance examination. (4 Marks)

d) Let X be a random variable having the density function given by

$$f(x) = \begin{cases} \frac{x^2}{9}, & 0 < x < 3 \\ 0, & otherwise \end{cases}$$

Find the variance of the random variable $g(x) = 2x + 3$ (6 Marks)

e) The probability of a successful outcome in a Binomial experiment is 0.4. Suppose 10 independent trials of the experiment are made. Let X be a random variable representing the number of successes. Obtain the following;

i) $P(X \geq 3)$ (4 Marks)

ii) $P(8 \leq X < 10)$ (3 Marks)

f) The mean number of flaws per metre of a cotton fabric is a random variable X that follows a Poisson distribution with parameter $\lambda = 2$. From a roll of cotton fabric, 5 metres are cut. Obtain the probability that there will be at least 1 flaw in the roll. (4 Marks)

QUESTION TWO (20 MARKS)

a) Batches that consist of 50 coil springs from a production process are checked for quality. The mean number of coil springs that do not meet the standard quality is found to be five. Assume that the number of coil springs in a batch that do not meet the required quality is denoted by X and X is a binomial random variable. Obtain

i. n and p (2 Marks)

ii. $P(X \leq 2)$ (2 Marks)

iii. $P(X \geq 49)$ (2 Marks)

iv. $P(2 < X \leq 48)$ (2 Marks)

b) A survey of 100 people is conducted. Among the 100 people 65 take tea, 40 take coffee, 70 take soda, 30 take both tea and coffee, 40 take both tea and soda, 25 take both coffee and soda while 20 take all the three drinks. A person is chosen at random out of the 100.

i. Represent this information on a Venn diagram (3 Marks)

Obtain the probability that the person chosen takes

ii. Only one of the drinks (2 Marks)

iii. At least one of the drinks (2 Marks)
iv. Exactly two of the drinks (3 Marks)
v. Tea only or Coffee (2 Marks)

QUESTION THREE (20 MARKS)

a) Let the random variable X represent the number of defective parts of a machine. When 3 parts are sampled from a production line and tested the following probability distribution of X is obtained

X	0	1	2	3
$f(x)$	0.3	0.25	t	0.15

If the the mean of the distribution is 1.1, obtain the value of t hence Variance (X) (7 Marks)

b) The number of bacterial colonies on a petri dish can be modelled by a Poisson distribution with average number 2.5 per $cm^{2.}$ Find the probability that

i. In $1cm^{2.}$ there are no bacterial colonies (3 Marks)
ii. In $2cm^{2.}$ there are more than three bacterial colonies (5 Marks)
iii. In $4cm^{2.}$ there are six or 9 bacterial colonies (5 Marks)

QUESTION FOUR (20 MARKS)

a) In a computer game the probability that the player hits the target is 0.4 for each independent attempt. Find
   i. The probability that he hits the target for the first time on the fourth attempt. (2 Marks)
   ii. The probability that he hits the target in more than four attempts. (3 Marks)
   iii. The mean and variance of the number of attempts to hit the target. (3 Marks)

b) Let X be a continuous random variable with probability distribution function given by

$$f(x) = \begin{cases} K(x+3), & -2 < x < 2 \\ 0, & otherwise \end{cases}$$

   i. Obtain K (4 Marks)
   ii. Compute $E(X)$ and Var(X) (8 Marks)

QUESTION FIVE (20 MARKS)

a) The number of shirts sold in a week by a famous boutique is normally distributed with a mean of 2080 and a standard deviation of 50. Estimate
   i. The probability that in a given week fewer than 2000 shirts are sold. (4 Marks)
   ii. The number of weeks in a year that between 2060 and 2130 shirts are sold (5 Marks)
   iii. The range of shirts sold in a week that represent the middle 95%. (5Marks)

b) The probabilities of events A and B are $P(A)$ and $P(B)$ respectively. $P(A) = 5/12$, $P(A \cup B) = q$, $P(A \cap B) = 1/6$ Find in terms of q

i. $P(B)$ (2 Marks)

ii. $P(A/B)$ (2 Marks)

iii. If A and B are independent find the value of q (2 Marks)