JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

2022/2023 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2109

COURSE TITLE: INTRODUCTION TO PROBABILITY THEORY

EXAM VENUE: **STREAM: ACTUARIAL SCIENCE**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question One Compulsory (30mks)

a) Briefly explain the meaning of the following terms as used in Probability (8marks)
   i) Sample Space
   ii) Sample Point
   iii) Probability
   iv) Exhaustive Events

b) If A^C *is the complement of event* A, *prove that* $P(A^C) = 1 - P(A)$ (5marks)

c) The Probability that John passes a Maths exam is 4/5 and that he passes a Chemistry exam is 5/6. If the probability that he passes both exams is 3/4, find the probability that he will pass at least one exam. (5marks)

d) In a large metropolitan area, the probability of a family owning a colour T.V , a computer or both 0.86, 0.35 and 0.29 respectively. What is the probability that a family chosen at random during a survey will own a colour T.V and/or a computer? Given that the family chosen at random during a survey owns a colour T.V, what is the probability that it will own a computer? (5marks)

e) A group of 50 people was asked which of the three novels they read A, B or C. the results showed that 25 read A, 16 read B, 14 read C, five read both A and C while 2 read all the three. If a person is chosen at random from these group, find the probability that he

i) Reads A only (2marks)
ii) Reads only one of the novels (2marks)
iii) Read at least one of the novels (3marks)

Question Two (20mks)

a) At a certain assembly plant, three machines make 30%, 45%, and 25%, respectively, of the products. It is known from the past experience that 2%, 3% and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected.

i)What is the probability that it is defective? (5marks)

ii) If a product were chosen randomly and found to be defective, what is the probability that it was made by machine 3? (5marks)

b) Of the customers at a gas station, 70% use regular gas, and 30% use diesel. Of the customers who use regular gas, 60% will fill the tank completely, and of those who use diesel, 80% will fill the tank completely.

i)What percent of all customers will fill the tank completely? (5marks)

ii) If a customer has filled up completely, what is the probability it was a customer buying diesel? (5marks)

Question Three (20mks)

a) Let X be a random variable with PDF given by $f_x(x) = \begin{cases} cx^2 & x \leq 1 \\ 0, otherwise \end{cases}$

i) Find the constant c. (4marks)

ii) Find E(X) and Var(X). (6marks)

iii) Find P(X≥$\frac{1}{2}$) (4marks)

b) Let X be a continuous random variable with PDF

$f_X(x) = \begin{cases} 4x^3 & 0 < x \leq 1 \\ 0, otherwise \end{cases}$

Find P(X≤$\frac{2}{3}$|X>$\frac{1}{3}$) (6marks)

Question Four (20mks)

Let X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} 0.1 \quad for\ x = 0.2 \\ 0.2\ for\ x = 0.4 \\ 0.2\ for\ x = .05 \\ 0.3 \quad for\ x = 0.8 \\ 0.2 \quad for\ x = 1 \\ otherwise \end{cases}$$

c) Find RX, the range of the random variable X. (5marks)

d) Find P(X≤0.5)P(X≤0.5). (5marks)

e) Find P(0.25<X<0.75) (5marks)

f) Find P(X=0.2|X<0.6). (5marks)

Question Five (20mks)

You take an exam that contains 20 multiple-choice questions. Each question has 4 possible options. You know the answer to 10 questions, but you have no idea about the other 10 questions so you choose answers randomly. Your score X on the exam is the total number of correct answers.

i)Find the PMF of X. (10marks)

ii)What is P(X>15) (10marks)