



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL

4TH YEAR SPECIAL RESITS – 2016

MAIN REGULAR

COURSE CODE: SMA 200

COURSE TITLE: CALCULUS II

EXAM VENUE:

STREAM: (BSc. Actuarial)

DATE:

EXAM SESSION:

TIME: 2.00 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) If $y' = f'(x) = 2x^2 + x - 1$ and $f(0) = 0$, then determine a relation in y and x (4 marks)

b) Evaluate the integral

$$\int \sqrt{1 + \sin 2x} \, dx \quad (5 \text{ marks})$$

c) Verify by differentiation that the formula is correct:

$$\int \frac{1}{\sqrt{a^2 + x^2}} \, dx = \log \left| x + \sqrt{a^2 + x^2} \right| + C \quad (6 \text{ marks})$$

d) Evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{2x}{(x^2 + 1)^2} \, dx \quad (5 \text{ marks})$$

e) Find the length of the curve $x = \left(\frac{y^{3/2}}{3} \right) - y^{1/2}$ from $y = 1$ to $y = 9$ (5 marks)

f) Determine whether the following series converges or diverges

$$\sum_{n=1}^{\infty} \frac{n2^2(n+1)!}{n^2!} \quad (5 \text{ marks})$$

QUESTION TWO (20 marks)

a) Evaluate the integral

$$\int \frac{x}{1 - x^2 + \sqrt{1 - x^2}} \, dx \quad (6 \text{ marks})$$

b) Evaluate

$$\int \frac{x^3}{\sqrt{1 - x^8}} \, dx \quad (4 \text{ marks})$$

c) By making the appropriate substitution for u in the integral below:

(i) Express the integral in terms of u . (3 marks)

(ii) Evaluate the integral as function of x . (3 marks)

$$\int_1^3 \frac{2x - 1}{(x + 1)^4} \, dx$$

d) Evaluate the integral

$$\int_0^{\pi/4} \frac{1}{\sin \theta + \cos \theta} \, d\theta \quad (5 \text{ marks})$$

QUESTION THREE (20 marks)

- a) Determine the value of the integral

$$\int_2^3 \frac{1}{3-2x-x^2} dx \quad (5 \text{ marks})$$

- b) Evaluate the integral

$$\int \frac{x^3 + 4x^2 - x}{(x+2)(x+1)} dx \quad (8 \text{ marks})$$

- c) Integrate by parts

$$\int e^{ax} \sin bx \, dx \quad (7 \text{ marks})$$

QUESTION FOUR (20 marks)

- a) Find the volume of the solid generated by revolving the region bounded by the curve

$$y = 4 - x^2 \text{ and line } y = 2 - x \text{ about the } x\text{-axis. (7 marks)}$$

- b) Determine the area of the surface generated by revolving the curve

$$y = \sqrt{2x - x^2}, \quad 0.5 \leq x \leq 1.5 \text{ about the } x\text{-axis. (6 marks)}$$

- c) Find the area of the region enclosed by the line $x + y^2 = 3$ and the curve $4x + y^2 = 0$. (7 marks)

QUESTION FIVE (20 marks)

- a) Evaluate $\int_0^1 e^{x^2} dx$ by Simpson's rule taking ten intervals (5 marks)

- b) Find a power series for the logarithmic function

$$L(x) = \ln(1+x)^3 \quad (6 \text{ marks})$$

- c) Show that the Taylor series about $x = 0$ for the function $f(x) = \cos x$ is

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad (5 \text{ marks})$$

- d) Evaluate the following integral

$$\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx \quad (4 \text{ marks})$$