



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR EXAMINATION FOR DEGREE IN BED SCI.
AND BED ARTS
SPECIAL RESIT EXAM 2020/2021
MAIN CAMPUS (BONDO)

COURSE CODE: SMA 403
COURSE TITLE: Topology I
EXAM VENUE:
STREAM: BED SCI. AND BED ARTS

DATE..... EXAM SESSION

TIME: 2 HOURS

Instructions:

- 1. Answer all questions in Section A and any other 2 questions in Section B**
- 2. Candidates are advised not to write on the question paper**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room**

SECTION A COMPULSORY (30 MARKS)

QUESTION ONE (30 MARKS)

- a) Define the following terms: (6 marks)
- i) Metric space
 - ii) Topological space
- b) Prove that $(A \cap B)^c = A^c \cup B^c$ (4 marks)
- c) Given the set $U = \{1,2,3,\dots,9\}$, $A = \{2,4,6,8\}$ and $B = \{1,3,7,9\}$. Find (3 marks)
- i) $A \Delta B$
 - ii) $A \cap B$
 - iii) A^c
- d) Prove that the empty set is unique (3 marks)
- e) Show that the function $d : X \times X \rightarrow R$ defined as $d(x, y) = \begin{cases} 0, & x=y \\ 1, & x \neq y \end{cases} \forall x, y \in X$ is a metric on X . (8 marks)
- f) Prove that the union of arbitrary family of open sets is open. (6 marks)

SECTION B

QUESTION TWO (20 MARKS)

- a) Given $X = \{1, 2, 3, 4, 5\}$ $\tau = \{X, \emptyset, \{5\}, \{2,3\}, \{2,3,5\}, \{1,2,3,4\}\}$ $A = \{5\}$, $B = \{3, 4\}$
Find (i). \bar{A} (2 marks)
(ii). Boundary of B (4 mark)
(iii). Limit points of A. (4 marks)
- b) Let (X, τ) be a topological space and $A \subseteq X$. Prove that A is closed if and only if it contains all its limit points. (10 marks)

QUESTION THREE (20 MARKS)

- a) Let $X = \{1,2,3\}$ and $\tau = \{X, \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$. Show that τ is a topology on X . (5 marks)
- b) Let X be a non-void set and τ_1, τ_2 be topologies on X . Show that $\tau_1 \cap \tau_2$ is a topology on X . (5 marks)
- c) Let $X = \{1,2,3\}$ $\tau = \{\emptyset, X, \{1\}, \{1,2\}, \{2,3\}\}$ and $A = \{1,3\}$. Find the $\text{Int}(A)$. (5 marks)
- d) Show that every convergent sequence in T_2 -space converges to a unique limit. (5 marks)

QUESTION FOUR (20 MARKS)

- a) Show that the indiscrete topological space is T_0 space. (4 marks)

- b) Prove that the topological space (X, τ) is a T_1 space if and only if every singleton subset of X is closed. (6 marks)
- c) Show $d: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ defined by $d(x, y) = |x^2 - y^2|$ is a metric on \mathbb{R} . (5 marks)
- d) Let $X = \{1, 2, 3, 4, 5\}$, $\tau = \{X, \emptyset, \{5\}, \{2, 3\}, \{2, 3, 5\}, \{1, 2, 3, 4\}\}$ and $A = \{3, 4, 5\}$. Find all the accumulation points of A . (5 marks)

QUESTION FIVE (20 MARKS)

- a) Define a topology and suppose that $X = \{a, b, c, d, e, f\}$ and $\tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e, f\}\}$. Is τ a topology on X ? (5 marks)
- b) Let (X, τ) be a topological space and $A \subseteq X$. If $X = \{1, 2, 3, 4\}$, $A = \{1, 2\}$, $B = \{3, 4\}$,
 $\tau = \{\emptyset, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}, X\}$.
 Find
- (i). $\text{Int}(A)$ (1 mark)
- (ii). $\text{Ext}(A)$ (2 marks)
- (iii). $\text{Bdy}(A)$ (2 marks)
- (iv). $\text{Bdy}(B)$ (5 marks)
- c) Show that the indiscrete topological space is indeed a topological space. (5 marks)