



JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE
UNIVERSITY EXAMINATION FOR THE BACHELORS DEGREE
1ST YEAR 1ST SEMESTER 2013/2014 ACADEMIC YEAR
CENTRE: MAIN

COURSE CODE: SMA 101

COURSE TITLE: ANALYTICAL GEOMETRY

EXAM VENUE: AH

STREAM: (BSc. Actuarial, Bed, B Sc)

DATE: 25/4/2014

EXAM SESSION: 2.00 – 4.00 PM

TIME: 2 HOURS

Instructions:

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY)

- a) Use the third order matrix determinant to determine the equation of a line passing through the points (2, 4) and (6, -4) giving your answer in double intercept form, hence declaring the intercepts. (4 marks)
- b) Determine the equation of a plane passing through three points A(3,3,3), B(6,6,3) and C(3,6,6) (4 marks)
- c) A line L_1 has an equation $y = 4x + 6$. Calculate the acute angle between L_1 and L_2 which passes through (0,2) and (2,-2) (3 marks)
- d) Calculate the area of a circle which passes through (3, 2), (-14, -5) and (10, -5). (Take $f = 3.142$) (7 marks)
- e) Convert the following Cartesian coordinates in to polar coordinates with negative coordinates only
(i) (-6, 10) (ii) (-8, -1) (4 marks)
- f) A conic section has the equation $16y^2 - 9x^2 - 96y = 0$
(i) Identify the conic section. (1 mark)
(ii) Determine the eccentricity. (3 marks)
(iii) Suppose the equation of the conic section is obtained by a translation of axes through an angle $r = 60^\circ$, find the original equation before the translation. (5 marks)

QUESTION TWO (20 marks)

- a) Identify the conic sections given below
(i) $2x^2 - 4xy + 4y^2 - 24x + 6y = 0$
(ii) $y^2 - 2xy + 3x^2 = 25$
(iii) $xy = y^2 + 4$ (3 marks)
- b) An analytic geometry student came across a sketch of a parabola. He identified three points on the parabola (0, -4), (2, 8) and (-2, 0). Given that the axis of the parabola is parallel to the $y -$ axis.
a. What is the equation of the parabola in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A or C is equal to zero. (7 marks)
(iii) Determine the focus and vertex of the parabola. (2 marks)
- b. Find the equation of the directrix and axis of the parabola. (2 marks)
- c) A triangle has vertices A(-5, 2), B(0, 3) and C(5, 7). Calculate the value of the largest angle in the triangle hence state whether it is acute or obtuse. (6 marks)

QUESTION THREE(20 marks)

a) The equation of an ellipse is given by $12x^2 + 120x + 3y^2 - 18y = -219$

Find on the xy plane

- (i) The centre of the ellipse (4 marks)
- (ii) The coordinates of the vertices (2 marks)
- (iii) The foci (2 marks)
- (iv) The eccentricity (1 mark)
- (v) The directrices (2 marks)
- (vi) The area of the ellipse (3 marks)

b) Consider the polar curve of a conic section given by $r = \frac{9}{3 - 6\cos\theta}$, sketch the curve hence determine the eccentricity, focus and equation of the directrices of the conic section. (6 marks)

QUESTION FOUR (20 marks)

a) Determine the centre and radius of a circle whose equation is

$$x^2 - 10x + y^2 + 12y - 39 = 0 \quad (4 \text{ marks})$$

b) Sketch the following polar curves and give its name.

$$r = 1 + 2\cos\theta \quad (6 \text{ marks})$$

c) The equation of a hyperbola $x^2 - 4y^2 + 2x + 8y - 7 = 0$ Find the equation of the asymptotes of the hyperbola. (6 marks)

d) Determine the distance between the lines $y = 2x + 4$ and $4x - 2y - 12 = 0$ (4 marks)

QUESTION FIVE (20 Marks)

a) Find the cartesian equation represented by the pairs of parametric equations given below.

- (i) $x = 3t + 1, y = t^2$
- (ii) $x = \cos 2\theta, y = \sin 3\theta$ (6 marks)

b) (i) A second degree curve is represented by the equation

$$17x^2 + 2xy + 17y^2 - 23\sqrt{2}x + 41\sqrt{2}y - 71 = 0.$$

By eliminating the cross product term give the new equation of the conic section on the new $x'y'$ plane and state the equation of its axis. (8 marks)

(ii) On the new $x'y'$ plane determine the foci, the eccentricity and the directrices of the conic section. (6 marks)