

**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE
AND TECHNOLOGY**
SPECIAL RESIT EXAMINATION-APRIL/MAY 2016
SMA 300: REAL ANALYSIS I

INSTRUCTION: Attempt question one (**COMPULSORY**) and any other **TWO** questions only.

QUESTION ONE (COMPULSORY) [30 MARKS]

- (a). Define a neighbourhood in a metric space (2 marks)
- (b). State 4 properties of the integral of a step function. (4 marks)
- (c). Determine the monotonicity of the sequence $\frac{1}{n}, \forall n \in \mathbb{N}$. (5 marks)
- (d). Show that any convergent sequence is bounded (5 marks)
- (e). Differentiate between Limit superior and Limit inferior. (4 marks)
- (f). Show that if $f : [a, b] \rightarrow \mathbb{R}$ is continuous, then f is Riemann integrable. (5 marks)
- (g). Define the following terms: Riemann Integrable function, Closed set, Accumulation point, Exponential function and Compactness. (5 marks)

QUESTION TWO [20 MARKS]

- (a). State the Lipschitz inequality. (2 marks)
- (b). State and prove Bolzano-Weierstrass Theorem for sequences. (9 marks)
- (c). Prove that no rational x satisfies the equation $x^3 = x + 7$. (9 marks)

QUESTION THREE [20 MARKS]

- (a). Show that the set $C_R[0, 1]$ of all real-valued continuous functions defined on $[0, 1]$ is a metric space with respect to the metric defined as $d(f, g) = \sup\{|f(x) - g(x)|; x \in [0, 1]\}$ where $f, g \in C_R[0, 1]$. (10 marks)
- (b). Show that (X, \mathfrak{D}) is a topological space where \mathfrak{D} is the class of all subsets of X . (10 marks)

QUESTION FOUR [20 MARKS]

- (a). Define a pseudo metric space. (3 marks)
- (b). Describe a characteristic function. (5 marks)
- (c). Describe Cauchy's Root Test and hence determine the convergence of $x_n = \sqrt{n^4 + 1} - \sqrt{n^4 - 1}$. (12 marks)

QUESTION FIVE [20 MARKS]

- (a). Define uniform continuity of a function. (2 marks)
- (b). Determine uniform continuity of $f(x) = x^{-1}$ on $(0, \infty)$. (8 marks)
- (c). Prove that every infinite set has a countable subset. (10 marks)